

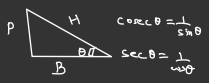
1. Trigonometric Ratios :

$i) \sin \theta = \frac{P}{H}$
 $ii) \cos \theta = \frac{B}{H}$
 $iii) \tan \theta = \frac{P}{B}$
 $iv) \cot \theta = \frac{B}{P}$
 $v) \operatorname{cosec} \theta = \frac{H}{P}$
 $vi) \sec \theta = \frac{H}{B}$

$P \rightarrow$ Perpendicular
 $B \rightarrow$ Base
 $H \rightarrow$ Hypotenuse

$\tan \theta = \frac{\sin \theta}{\cos \theta}$
 $\cot \theta = \frac{\cos \theta}{\sin \theta}$

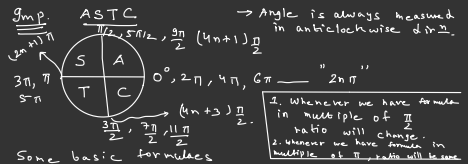
$P^2 + B^2 = H^2$



Note: It's always better to deal in $\sin$ or $\cos \theta$.

$\pi^\circ = 180^\circ$

$1. \pi = 180^\circ$
 $2. \frac{\pi}{2} = 90^\circ$
 $3. \frac{\pi}{4} = 45^\circ$
 $4. \frac{\pi}{3} = 60^\circ$
 $5. \frac{\pi}{6} = 30^\circ$
 $6. 2\pi = 360^\circ$



$1. \sin(\pi/2 - \theta) = +\cos \theta$
 $2. \cos(\pi/2 + \theta) = -\sin \theta$
 $3. \tan(\pi/2 - \theta) = +\cot \theta$
 $4. \sin(\pi/2 + \theta) = +\cos \theta$
 $5. \sec(\pi/2 + \theta) = -\operatorname{cosec} \theta$
 $6. \tan(\pi + \theta) = -\tan \theta$

$8. \cos(100\pi + \theta) = +\cos \theta$

$9. \cot(35\pi - \theta) = -\cot \theta$

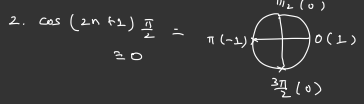
$10. \operatorname{cosec}(14\pi - \theta) = -\operatorname{cosec} \theta$

Values of allied angles

$1. \cos\left(\frac{4\pi}{3}\right) \rightarrow$ Think about a multiple of 2π closest to $\frac{4\pi}{3}$.
 $= \cos\left[\frac{12\pi}{3} - \frac{\pi}{3}\right] = \cos[4\pi - \pi/3] = +\cos \pi/3 = 1/2$

Note:

$1. \sin(n\pi) = 0$
 $2. \cos(2n\pi) = 1$



$3. \cos(2n\pi) = 1$
 $4. \cos(2n+1)\pi = -1$
 $5. \tan(2n+1)\pi = 0$
 $6. \cot(n\pi) = \frac{\cos n\pi}{\sin n\pi} = \frac{1}{0}$
 $7. \operatorname{cosec}(4n+1)\pi/2 = 1$

Note

C-D formulae's

$1. \sin C + \sin D = 2 \sin \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$
 $2. \sin C - \sin D = 2 \cos \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$
 $3. \cos C + \cos D = 2 \cos \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$
 $4. \cos C - \cos D = -2 \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$

Don't forget to apply the sign.

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Golden formulae's

$1. 1 + \cos 2\theta = 2 \cos^2 \theta$
 $2. 1 - \cos 2\theta = 2 \sin^2 \theta$

Formulae's for "-0"

$\rightarrow \sin(-\theta) = -\sin \theta$
 $\rightarrow \cos(-\theta) = \cos \theta$
 $\rightarrow \tan(-\theta) = -\tan \theta$

If $A+B = 45^\circ$

then $(1 + \tan A)(1 + \tan B) = 2$

Formulae for multiple of angles.

$\sin 2A = \sin(A+A)$
 $= \sin A \cos A + \cos A \sin A$
 $\sin 2A = 2 \sin A \cos A$

Learn
angle π & 2π angle double
karna ho

$\sin 2A = \frac{2 \sin A \cos A}{\cos^2 A + \sin^2 A}$ Divide by $\cos^2 A$
 $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$ Learn
if $\sin 2A$ has to be converted in $\tan$

$\cos 2A = \cos^2 A - \sin^2 A$
 $= 1 - 2 \sin^2 A$
 $= 2 \cos^2 A - 1$
 $= \frac{1 - \tan^2 A}{1 + \tan^2 A}$

1. Sine of supplementary angles are equal.

cosine of supplementary angles differ by a "-" sign.

Formulae for A+B and A-B.

$1. \sin(A+B) = \sin A \cos B + \cos A \sin B$
 $2. \sin(A-B) = \sin A \cos B - \cos A \sin B$
 $3. \cos(A+B) = \cos A \cos B - \sin A \sin B$
 $4. \cos(A-B) = \cos A \cos B + \sin A \sin B$
 $5. \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$6. \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

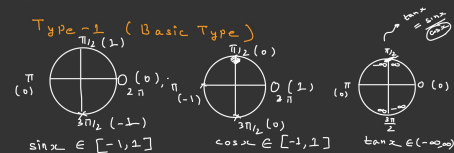
Formulaes for 30

- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
- $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
- $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Range of trigonometric functions.
 gmp. OR to find maximum and minimum values of y

$$y_{\min} = 1 \quad y \in [1, 7] \\ y_{\max} = 7$$

Type-1 (Basic Type)

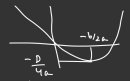


Type-2 Range of trigonometric functions convertible to quadratic expressions.

Range of Quadratic expressions:

Type-I $x \in \mathbb{R} \Rightarrow y = ax^2 + bx + c$

i) $a > 0$



$$y_{\max} = \infty \\ y_{\min} = -\frac{1}{4a} \text{ at } x = -\frac{b}{2a}$$

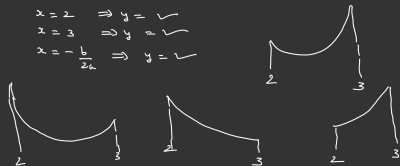
ii) $a < 0$



$$y_{\min} = -\infty \\ y_{\max} = \frac{1}{4a} \text{ at } x = -\frac{b}{2a}$$

Type-2 $x \in [2, 3]$

$$x=2 \Rightarrow y=1 \\ x=3 \Rightarrow y=1 \\ x=-\frac{1}{2a} \Rightarrow y=1$$



Find the range of $y = 2\cos^2 x - \sin x + 1$.

Sol 2 $y = 2\cos^2 x - \sin x + 1$
 $y = 2(1 - \sin^2 x) - \sin x + 1$

$$\sin x = t$$

$$y = 2 - 2t^2 - t + 1$$

$$y = -2t^2 - t + 3$$

$$t \in [-1, 1]$$

$$t = -1 \Rightarrow y = -2 + 1 + 3 = 2 = y$$

$$t = 1 \Rightarrow y = -2 - 1 + 3 = 0 = y$$

$$t = -\frac{b}{2a} = -(-1)/2 = -1/2 \Rightarrow t = -1/4 \Rightarrow y = -1/8 + 1/4 + 3 = \frac{25}{8}$$

Type-3 Range of Trigonometric functions.

→ Range of expressions of the type

$$y = a \cos \theta + b \sin \theta$$

→ PTR.

i) $\theta \in \mathbb{R}$

ii) θ should be same

iii)

$$y_{\max} = \sqrt{a^2 + b^2}$$

$$y_{\min} = -\sqrt{a^2 + b^2}$$

Note Whenever we have an expression of the type $a \cos \theta + b \sin \theta$, always multiply and divide by $\sqrt{a^2 + b^2}$

Type-4 To find range of the expressions of the type

gmp.

$$y = a^2 \tan^2 \theta + b^2 \cot^2 \theta$$

$$y_{\max} = \infty$$

$$y_{\min} = 2ab$$

Concept

$$A.M. \geq G.M.$$

Some - More formulaes

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$

gmp.

- $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$
- $\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$
- $\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$
- $\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$

Golden Rule Whenever in complete mathematics

We have 2 terms of sin in multiplication or 2 terms of cos " " or 1 sin and 1 cos " "

Always divide and multiply by 2 to use these 4 formulae.

$$\text{e.g. } \frac{2 \sin A \sin B}{2} = \cos(A-B) - \cos(A+B)$$

$$\frac{2 \cos A \cos B}{2} = \cos(A+B) + \cos(A-B)$$

$$\frac{2 \sin A \cos B}{2} = \sin(A+B) + \sin(A-B)$$

$$1. \sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$$

$$2. \cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B$$

Conditional Identities

In a $\triangle ABC$ or $A+B+C = \pi$ (To click in exams)

Concepts

$$1. \sin(A+B) = \sin C$$

$$\text{Proof } \sin[A+B] = \sin[\pi - C] = \sin C$$

$$2. \cos(A+B) = -\cos C$$

$$\text{Proof } \cos[\pi - C] = -\cos C \quad A+B = \pi - C$$

$$3. \sin\left(\frac{A+B}{2}\right) = \cos \frac{C}{2}$$

$$4. \cos\left(\frac{A+B}{2}\right) = \sin \frac{C}{2}$$

$$\cos\left[\frac{\pi}{2} - \frac{C}{2}\right]$$

$$= \sin \frac{C}{2}$$

2 Golden Results in a triangle

gmp.

In a $\triangle ABC$

- $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$
- $\tan A/2 \cdot \tan B/2 + \tan B/2 \cdot \tan C/2 + \tan C/2 \cdot \tan A/2 = 1$

How to click in an exam.

$$\text{In a } \triangle ABC, \tan A \tan B \tan C$$

Trigonometric - Series

Type-1: Whenever we have a series in the form of sum of sine or cosine and angles are in A.P.

$$1) \sin \theta + \sin 2\theta + \sin 3\theta + \dots + \sin n\theta$$

$$= \frac{\sin \left(n \cdot \frac{1}{2} \text{ diff.} \right)}{\sin \left(\frac{1}{2} \text{ diff.} \right)} \cdot \frac{\sin \frac{1+L}{2}}$$

n = no. of terms
 diff = diff. of the angles
 I = first angle
 L = last angle

$$ii) \cos \theta + \cos 2\theta + \cos 3\theta + \dots + \cos n\theta$$

$$= \frac{\sin \left(n \cdot \frac{1}{2} \text{ diff.} \right)}{\sin \left(\frac{1}{2} \text{ diff.} \right)} \cdot \frac{\cos \frac{1+L}{2}}$$

Note ^{Imp.} While solving any type of trigonometric series, you should know how to play with supplementary and complementary angles.

Type-2

Whenever we have a series in the form of product of cosine and each angle is double of angle before it

$$\cos \theta \cdot \cos 2\theta \cdot \cos 2^2\theta \cdot \cos 2^3\theta \dots \cos 2^{n-1}\theta$$

$$= \frac{\sin (2^n \theta)}{2^n \sin \theta}$$

*** learn

n → No. of terms.
 θ → smallest angle.

Trigonometric Equations.

- Principle Solution
 - All values of variable " θ " lying in $[0, 2\pi]$
 - Generally it is represented by " α "
 - Generally we will take the least value while writing the principle solution.
- e.g. $\sin \theta = \frac{1}{2}$. Find the principle soln of θ .

$\theta = \pi/6 = \alpha$
 → General solution
 All values of the variable " θ "
 A solution which determines all the possible values of θ .

- $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha$
- $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$
- $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha$
- $\sin^2 \theta = \sin^2 \alpha$
- $\cos^2 \theta = \cos^2 \alpha \Rightarrow \theta = n\pi \pm \alpha$
- $\tan^2 \theta = \tan^2 \alpha$

Type - 1.

(Basic - Type)

Q1 Find the general solution of the equation given by $\sec^2 2x = 1 - \tan 2x$.

Soln

$$\sec^2 2x = 1 - \tan 2x$$

$$1 + \tan^2 2x = 1 - \tan 2x$$

$$\tan^2 2x + \tan 2x = 0$$

$$\tan 2x (\tan 2x + 1) = 0$$

$$\tan 2x = 0 \quad \text{OR} \quad \tan 2x = -1$$

$$2x = n\pi \quad \text{OR} \quad 2x = n\pi + \frac{3\pi}{4}$$

$$x = \frac{n\pi}{2} \quad \text{OR} \quad x = \frac{n\pi}{2} + \frac{3\pi}{8}$$

Type-2 Very easy

$\sin () = \cos ()$
 $\tan () = \cot ()$
 $\sec () = \csc ()$

Note Make use of complementary angle formula of class-10th

Q.6 Find general solution of

$$\sin 2x = \cos 5x$$

Soln

$$\sin 2x = \cos 5x$$

$$\Rightarrow \cos \left[\frac{\pi}{2} - 2x \right] = \cos 5x$$

$$\frac{\pi}{2} - 2x = 2n\pi \pm 5x$$

$$\Rightarrow \frac{\pi}{2} - 2x = 2n\pi + 5x \Rightarrow 7x = \frac{\pi}{2} - 2n\pi$$

$$\Rightarrow \frac{\pi}{2} - 2x = 2n\pi - 5x \Rightarrow 3x = 2n\pi - \frac{\pi}{2}$$

$$\Rightarrow x = \frac{2n\pi}{3} - \frac{\pi}{6}$$

$\cos \theta = \cos \alpha$
 $\theta = 2n\pi \pm \alpha$

Type-3

Solving equations of the type

$$a \cos \theta + b \sin \theta = c$$

θ must be same.

Step-I
 → Divide both sides by $\sqrt{a^2 + b^2}$
 → Assume $\frac{a}{\sqrt{a^2 + b^2}}$ or $\frac{b}{\sqrt{a^2 + b^2}}$ as $\cos \phi$ to
 Create the formula of $\cos (A+B)$
 OR
 $\cos (A-B)$.

Q.1 Find the general solution of the equation: $\sin x + \cos x = \sqrt{2}$

Soln

$$\sin x + \cos x = \sqrt{2}$$

It is an eqn. of the type $a \sin x + b \cos x = c$

→ Divide both sides by $\sqrt{2}$

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = 1$$

$$\sin \frac{x}{4} + \cos \frac{x}{4} = 1$$

$$\cos \left(x - \frac{\pi}{4} \right) = 1 \quad x = 0$$

$$x - \frac{\pi}{4} = 2n\pi$$

$$x = 2n\pi + \frac{\pi}{4}$$

Type-4 Imp.

When ever we have different types of expressions on L.H.S and R.H.S, we won't be able to manipulate. We always find range of both sides

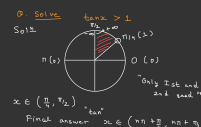
Note Expressions of the type

$$\frac{t + \frac{1}{t}}{t} \in (-\infty, -2] \cup [2, \infty)$$

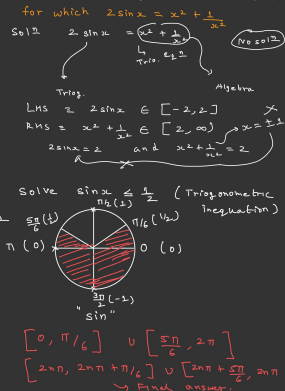
$t < 0$ $t > 0$

Trigonometric Inequalities (Imp.)

- Draw an ASTC chart of $\sin$ or $\cos$ or $\tan$.
- Mark the values on the boundaries.
- Mark the required value also on ASTC.
- Shade the region of the given inequality.
- Write the final answer by adding $2n\pi$ in $\sin$ or $\cos$ problems. $n\pi$ in $\tan$ problems



Q.3 Find all the possible values of x for which $2 \sin x = x + \frac{1}{x^2}$



Values to be Remembered

- $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$
- $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$
- $\tan 15^\circ = 2 - \sqrt{3}$
- $\cot 15^\circ = 2 + \sqrt{3}$
- $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$
- $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$
- $\tan 22.5^\circ = \sqrt{2}-1$
- $\cot 22.5^\circ = \sqrt{2}+1$

Kindly note that values of ratios starting with $\sqrt{2}$ are with 45° sign